Do tandem and triple axles really deserve their bad reputation?

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ABSTRACT: The SETRA Technical Guide to the Conception and Design of Roads (1994, France) considers that the impact of a tandem axle on a rigid pavement is equivalent to 12 single axles and that a triple axle is equivalent to 113 single axles. Upon reflection these Figures are astounding. Without a doubt one of the aims of vehicle builders in moving from the tandem to the triple axle was to reduce the stresses, for equal loads, induced to a vehicle chassis. This prompted us to ask why the effect on the road is so very much the opposite. In contrary to what happens between the two axles of 2 successive vehicles, the bending stresses at the base of the pavement do not entirely abate between the individual components of a tandem or triple axle. On the basis of this observation we have been able to show that the detrimental impact of tandem or triple axles is markedly lower than has been thought in the past. The design process needs only take account of the most stressing component of the multiple axle. In this case, the allowable number of axles could be more or less doubled.

KEY WORDS: Concrete pavement Design, tandem axles, triple axles

1. ANALYSIS BASED ON A MULTI-LAYER MODEL

Let us consider a concrete pavement structure with characteristics as presented in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>H (mm)</th>
<th>E (N/mm²)</th>
<th>μ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavement</td>
<td>200</td>
<td>30000</td>
<td>0.20</td>
</tr>
<tr>
<td>Base course</td>
<td>200</td>
<td>500</td>
<td>0.50</td>
</tr>
<tr>
<td>Sub-base</td>
<td>400</td>
<td>200</td>
<td>0.50</td>
</tr>
<tr>
<td>Soil</td>
<td></td>
<td>30</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Then impose the structure to a single axle, a tandem axle, and a triple axle moving parallel to the x-axis. Each individual axle represents a total load of 100 kN. The tyre pressure is 0.7 N/mm². The tandem and distance(s) of an axle is 0.5 meter. The bending stresses $\sigma_y$ at the base of the concrete pavement and in the axis of the loads are given in Table 2.
Table 2: Bending stresses in N/mm²

<table>
<thead>
<tr>
<th>Single Axle</th>
<th>Tandem Axle (2 × 100 kN)</th>
<th>Triple Axle (3 × 100 kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axle 1</td>
<td>Axle 1</td>
<td>Axle 1</td>
</tr>
<tr>
<td>1.457</td>
<td>1.729</td>
<td>1.729</td>
</tr>
<tr>
<td>Axle 2</td>
<td>Axle 1</td>
<td>Axle 2</td>
</tr>
<tr>
<td></td>
<td>1.729</td>
<td>1.819</td>
</tr>
<tr>
<td>Axle 3</td>
<td>Axle 2</td>
<td>Axle 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.002</td>
</tr>
</tbody>
</table>

These stresses are plotted in Figure 1 in relation to their distance to the axis of the loads concerned.

![Bending Stresses as a function of the distance to the load](image)

Figure 1: Bending Stresses as a function of the distance to the load

If we now apply the fatigue law used in the Netherlands (CROW, 1999), which has the virtue of accounting for minimum and maximum stress levels.

\[
\log N = 13 \times \frac{1 - \frac{\sigma_{\text{bending max}}}{\sigma_{\text{rupture}}}}{1 - 0.75 \frac{\sigma_{\text{bending min}}}{\sigma_{\text{rupture}}}}
\]

The allowable number of single axles is obtained from:

\[
\log N_{\text{single}} = 13 \times \left[1 - \frac{1.457}{6}\right] = 9.8432 \Rightarrow N_{\text{single}} = 6.97 \times 10^9
\]

The allowable number of tandem axles is obtained from:

\[
\log N_{\text{single}} = 13 \times \left[1 - \frac{1.729}{6}\right] = 9.2538 \Rightarrow N_{\text{single}} = 1.79 \times 10^9
\]

\[
N_{\text{tandem}} = \frac{1.79 \times 10^9}{2} = 0.895 \times 10^9
\]
For the structure under consideration the detriment of a tandem axle corresponds to \( \frac{6.97 \times 10^9}{0.895 \times 10^9} = 7.79 \) single axles and not to 2 single axles.

The allowable number of triple axles is obtained from:

\[
\log N_{\text{axle1}} = \log N_{\text{axle3}} = 13 \left[ 1 - \frac{1.819}{6} \right] = 9.0588 \quad \Rightarrow \quad N_{\text{axle1}} = 1.15 \times 10^9
\]

\[
\log N_{\text{axle2}} = 13 \left[ 1 - \frac{2.002}{6} \right] = 8.6623 \quad \Rightarrow \quad N_{\text{axle2}} = 0.460 \times 10^9
\]

\[
N_{\text{triple}} = \frac{1}{3} \times \frac{1}{3} \left[ \frac{2}{1.15 \times 10^9} + \frac{1}{0.46 \times 10^9} \right] = 0.256 \times 10^9
\]

In this case the detriment of a triple axle corresponds to \( \frac{6.97 \times 10^9}{0.256 \times 10^9} = 27.2 \) single axles and not 3 single axles.

We consider these values to be excessive because they do not take account of the fact that the bending stress does not return to zero in the zones between the axles, and that as a result the fatigue sustained by the structure is thus reduced.

### 2. ANALYSIS OF THE FATIGUE LAW

Let us consider Figure 1.

It may be observed that in the case of the single axle, the stress varies from a value of zero at about 3000 mm ahead of the maximum value of 1.457 N/mm² under the load to return to zero about 3000 mm beyond the load.

Consider the curve linking the successive values for the bending stress as an influence line. Its similarity to that produced during a laboratory fatigue test on a cylinder is striking, with the stressing varying more or less in a sinusoidal way between 0 and 1.457 N/mm².

The value of the bending stress to be introduced into the fatigue law is the difference between the maximum (1.457 N/mm²) and the minimum (0.0 N/mm²) before and after the application of the load of 1.457 N/mm².

Let us now consider the case of a cylindrical sample that firstly has been subjected to an unvarying stress (0.5 N/mm²), followed by a sinusoidal stress varying between 1.457 N/mm² and the initial value of 0.5 N/mm². The value of the bending stress to be entered into the fatigue law is now equal to the maximum value (1.457 N/mm²) minus the minimum value (0.5 N/mm²) or 0.957 N/mm².

The difference between the two cases is pronounced.

A similar situation is encountered in the case of a tandem or of a triple axle.

Let us consider Figure 2, which shows the course of the bending stress when subjected to the passage of a tandem axle.

The stress is zero at about 3500 mm ahead of the first axle, and reaches a maximum value (1.729 N/mm²) directly under the first axle, falls to a local minimum halfway between the 2
axles (1.152 N/mm²), and reaches a new maximum under the second axle before falling back to zero roughly 3500 mm beyond the second axle.

What stress value should be used in the fatigue law? If one determines it on the left of the first axle this value is equal to

\[ 1.729 \text{ N/mm}^2 - 0.0 \text{ N/mm}^2 = 1.729 \text{ N/mm}^2 \]

On the other hand, if one determines it at the right of the first axle the value becomes:

\[ 1.729 \text{ N/mm}^2 - 1.152 \text{ N/mm}^2 = 0.577 \text{ N/mm}^2 \]

The difference is significant.

Current methods take account of the two maximum stresses (1.729 N/mm²) and determine the number of allowable axles by Miner’s law. While admittedly erring on the safe side, they do so to excess, and the extent of this error is difficult to estimate. We propose the following solution. Let us break down the curve into two sections as shown in Figures 3 and 4.
Figure 3: Maximum stress in the axis of the load

Figure 4: Minimum stress between the loads

Figure 5 is obtained by transforming Figure 4 in such a way that the usual influence line is obtained. This does not present any problem at all for the concrete, but makes it easier to represent the magnitude of the fatigue bending stresses acting on the structure.
Let $N_1$ be the allowable number of axles under the load as shown in Figure 3.

$$\log N_1 = 13 \left[ 1 - \frac{1.729}{6} \right] = 9.2538 \quad \Rightarrow \quad N_1 = 1.79 \times 10^9$$

Let $N_2$ be the allowable number of axles under the load as shown in Figure 5.

$$\log N_2 = 13 \left[ 1 - \frac{1.729}{6} \right] = 10.8106 \quad \Rightarrow \quad N_2 = 64.6 \times 10^9$$

Application of Miner's law allows us to determine the allowed number of single axles on the structure

$$N_{\text{single}} = \frac{1}{2} \left[ \frac{1}{1.79 \times 10^9} + \frac{1}{64.6 \times 10^9} \right] = 3.48 \times 10^9$$

The corresponding number of tandem axles is equal to $1.74 \times 10^9$.

The detrimental effect of a 200 kN tandem axle thus corresponds to $\frac{6.97 \times 10^9}{1.74 \times 10^9} = 4.01$ single 100 kN axles, or is equal to a damage that is two times less than that indicated by the initial calculation.

For a triple axle, the stress is zero at about 3700 mm ahead of the first axle, reaches its maximum (1.819 N/mm²) in the axis of the first axle, falls back to a local minimum half way between the first and second axles (1.303 N/mm²), rises to an absolute maximum (2.002 N/mm²) in the axis of the central axle, falls back to 1.303 N/mm² half way between the
second and third axles, rises to a maximum again (1.819 N/mm²) in the axis of the third axle and finally disappears at 3700 mm beyond the third axle.

We use the same method adopted for the tandem axle.

Let \(N_1\) be the allowable number of axles under the maximum load (2.002 N/mm²). We find:

\[
\log N_1 = 13 \cdot \left[ 1 - \frac{2.002}{6} \right] = 8.6623 \quad \Rightarrow \quad N_1 = 0.460 \times 10^9
\]

Let \(N_2\) be the allowed number of axles under the minimum load:

\[
\log N_2 = 13 \cdot \left[ 1 - \frac{1.819}{6} \frac{1.303}{6} - \frac{1.75}{6} \right] = 10.8214 \quad \Rightarrow \quad N_2 = 66.3 \times 10^9
\]

Application of Miner’s law allows us to determine the allowable number of single axles on the structure

\[
N_{\text{single}} = \frac{1}{3} \left[ \frac{1}{0.460 \times 10^9} + \frac{2}{66.3 \times 10^9} \right] = 1.36 \times 10^9
\]

The corresponding number of triple axles is 0.453 \times 10^9.

In this case the detriment of the triple axle of 300 kN equals that of \(\frac{6.97 \times 10^9}{0.453 \times 10^9} = 15.4\) single 100 kN axles instead of the 27.2 reported earlier.

Moreover we find that the damage of a triple axle corresponds to that of \(\frac{1.74 \times 10^9}{0.453 \times 10^9} = 3.84\) tandem axles of 200 kN.

3. CONCLUSION

In Table 3 we compare the number \(N_1\) of allowable maximum loads to the allowed number of tandem and triple axles respectively:

<table>
<thead>
<tr>
<th>Type of axles</th>
<th>Tandem</th>
<th>Triple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum load</td>
<td>(1.79 \times 10^9)</td>
<td>(0.460 \times 10^9)</td>
</tr>
<tr>
<td>Combined axles</td>
<td>(1.74 \times 10^9)</td>
<td>(0.453 \times 10^9)</td>
</tr>
</tbody>
</table>

The values obtained are more or less the same for both tandem and triple axles. We propose acknowledging that these values are effectively the same and that the structure should therefore be designed so that only the total stress under one of the two axles should
be taken into account for tandem axles, and only the total stress under the central axle for triple axles.

Another approach is to calculate the total load which gives the same equivalent detrimental effect. Based on the conclusion above, Table 4 gives the total load which results in the same detrimental effect (i.e. reaches the same maximum stress) as two axles of 100 kN.

<table>
<thead>
<tr>
<th>Type of axles</th>
<th>Load (kN)</th>
<th>Maximum stress</th>
<th>Total load (for $\sigma_y = 1.729$ N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single</td>
<td>100</td>
<td>1.457</td>
<td>119 kN</td>
</tr>
<tr>
<td>Tandem</td>
<td>200</td>
<td>1.729</td>
<td>200 kN (2 × 100 kN)</td>
</tr>
<tr>
<td>Triple</td>
<td>300</td>
<td>2.002</td>
<td>259 kN (3 × 86.3 kN)</td>
</tr>
</tbody>
</table>

One can conclude that by using tandem or triple axles more goods can be transported, resulting in the same detrimental effect of the pavement.

REFERENCES
