Shear transfer and deflection ratio’s at a joint of a concrete slab

Dr. F. Van Cauwelaert and Mr. M. Stet
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Summary

At the 1993 Purdue Conference one of the authors presented an analytical solution for the computation of stresses in slabs submitted to edge loads eventually allowing for shear transfer at the edge (joint). However no mean was indicated for the practical evaluation of the amount of transferred shear, represented in the model by a factor $\gamma$ varying from 5 (no transfer) to 1 (full transfer).

At an informal seminar held in Brussels at the end of 1996, Ioannides presented unpublished research work performed at WES, under supervision of Westergaard, by Skarlatos in the late forties: a model giving a relation between the bending stresses and deflections at both sides of a joint.

In this paper, the authors link the two approaches in determining, with an extremely simple formula, the amount of transferred shear in function of the ratio between the deflections on both sides of the joint. Hence the ratio between the bending stress at the joint and in the middle of an infinite slab can easily be evaluated which is important for the design procedure of the slab itself as for the conception of the load transfer devices in the joint.

However the notion of load transfer differs considerably with that of Skarlatos. In his approach, the ratio of the bending stresses is a third degree function of the ratio of the deflections, while in this paper it comes out that the ratio of the bending stresses is strictly equal to the ratio of the deflections.
1. Introduction

At the 1993 Purdue Conference, Van Cauwelaert (1993) presented an analytical solution for the computation of stresses in slabs submitted to edge loads eventually allowing for shear transfer at the edge (joint). However no mean was indicated for the practical evaluation of the amount of transferred shear, represented in the model by a factor $\gamma$ varying from 0 (no transfer) to 1 (full transfer).

At an informal seminar held in Brussels at the end of 1996, Ioannides presented unpublished research work performed at WES, under supervision of Westergaard, by Skarlatos in the late forties: a model giving a relation between the deflections at both sides (loaded and unloaded) of a joint and a relation between the bending stresses. Together with M. Hammons, he published Transportation Research Record No. 1525 (Ioannides, 1996) on the same subject.

In this paper, the two approaches are linked in determining, with an extremely simple formula, the amount of transferred shear as a function of the ratio between the deflections on both sides of the joint. Hence the ratio between the bending stresses at the joint in comparison with their value in the middle of an infinite slab can be easily evaluated.

However the notion of load transfer at the joint differs considerably between the two approaches. In Ioannides paper the ratio between the bending stresses at both sides of the joint is a third degree function of the ratio of the deflections and varies with the shape of the load, while in this paper the ratio of the bending stresses is strictly equal to the ratio of the deflections. The reason of the difference, which should carefully been analyzed, lies probably in the different assumptions done about the evaluation of shear from the loaded side to the unloaded side of the joint. Full scale in situ testing would certainly provide for a better insight in the phenomenon.

2. General solution

The development of the solution is mathematical identical, for what concerns the expression of the load, as the 1993 solution. However similar boundary conditions as those of Skarlatos were taken into account.

The differential equation for the deflection of an infinite slab, with rigidity $D$, submitted to a load $p$ and resting on an elastic subgrade with reaction $q$ writes
\[
\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] + \frac{q}{D} = \frac{D}{D}
\]

Making Westergaard’s assumption \( q = kw \), with \( k \) the subgrade modules, (1) transforms into

\[
\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] + \frac{kw}{D} = \frac{D}{D}
\]

For a load uniformly distributed over a rectangular area \( 2a \times 2b \) and located at the origin of the coordinates, the solution of (2) is

\[
p = \frac{4p}{\pi^2} \int_0^\infty \int_0^\infty \cos(tx/l) \sin(ta/l) \cos(sy/l) \sin(sb/l) \frac{dsdt}{ts}
\]

\[
w = \frac{4p}{\pi^2} \int_0^\infty \int_0^\infty \cos(tx/l) \sin(ta/l) \cos(sy/l) \sin(sb/l) \frac{dsdt}{ts(t^4 + 2r^2s^2 + s^4 + 1)}
\]

where \( t^4 = D/k \).

If we want to impose some boundary conditions related to the geometry of the slab, the variables \( x \) and \( y \) have to be separated in (4).

This can be done by partially integrating (4) against \( t \), if the considered boundary is parallel to the \( y \)-axis, or against \( s \) if it is parallel to the \( x \)-axis.

We shall consider boundaries parallel to the \( y \)-axis: hence the integration of (4) yields

- for \( 0 \leq x \leq a \)

\[
w = \frac{p}{\pi k} \int_0^\infty \frac{\cos(sy/l) \sin(sb/l)}{s(s^4 + 1)} \left\{ 2 - e^{-\alpha(a-x)/l} \left[ \cos \frac{a-x}{l} \beta + s^2 \sin \frac{a-x}{l} \beta \right] - e^{-\alpha(x+a)/l} \left[ \cos \frac{a+x}{l} \beta + s^2 \sin \frac{a+x}{l} \beta \right] \right\} ds
\]

- for \( x \geq a \)

\[
w = \frac{p}{\pi k} \int_0^\infty \frac{\cos(sy/l) \sin(sb/l)}{s(s^4 + 1)} \left\{ e^{-\alpha(x-a)/l} \left[ \cos \frac{x-a}{l} \beta + s^2 \sin \frac{x-a}{l} \beta \right] - e^{-\alpha(x+a)/l} \left[ \cos \frac{x+a}{l} \beta + s^2 \sin \frac{x+a}{l} \beta \right] \right\} ds
\]

where

\[
\alpha = \left[ \frac{(s^4 + 1)^{1/2} + s^2}{2} \right]^{1/2} \quad \beta = \left[ \frac{(s^4 + 1)^{1/2} - s^2}{2} \right]^{1/2}
\]
3. Particular solution

Consider a slab with a joint at a distance $d$ (taken in the positive $x$-direction) from the origin. In order to express the boundary conditions at the joint, we need for complementary solutions of the homogeneous equation

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right] + \frac{k w}{D} = 0 \quad (7)$$

Two particular solutions apply at the left side (the loaded side) of the joint:

$$\frac{p}{\pi k} \int_0^\infty A(s) \frac{\cos(sy / l) \sin(sb / l)}{s} e^{\alpha s l} \cos(\beta x / l)ds = Aw_A \quad (8)$$

$$\frac{p}{\pi k} \int_0^\infty B(s) \frac{\cos(sy / l) \sin(sb / l)}{s} e^{\alpha s l} \sin(\beta x / l)ds = Bw_B \quad (9)$$

Two particular solutions apply at the right side (the unloaded side) of the joint:

$$\frac{p}{\pi k} \int_0^\infty C(s) \frac{\cos(sy / l) \sin(sb / l)}{s} e^{-\alpha s l} \cos(\beta x / l)ds = Cw_C \quad (10)$$

$$\frac{p}{\pi k} \int_0^\infty D(s) \frac{\cos(sy / l) \sin(sb / l)}{s} e^{-\alpha s l} \sin(\beta x / l)ds = Dw_D \quad (11)$$

4. Boundary conditions

We express that at the joint, thus for $x = d$:
- some deflection can exist at the right side of the joint (unloaded side);
- no moment transfer through the joint is possible;
- some shear transfer through the joint is possible.

Those are the boundary conditions considered by Skarlatos (1948).

Mathematically:
- The deflection right from the joint is a fraction of the deflection left:
  $$\delta [w + Aw_A + Bw_B] = Cw_C + Dw_D \quad (12)$$
- The moment left from the joint is zero:
  $$\left[ \frac{\partial^2}{\partial x^2} + \mu \frac{\partial^2}{\partial y^2} \right] [w + Aw_A + Bw_B] = 0 \quad (13)$$
- The moment right from the joint is zero:
  $$\left[ \frac{\partial^2}{\partial x^2} + \mu \frac{\partial^2}{\partial y^2} \right] [Cw_C + Dw_D] = 0 \quad (14)$$
- Shear right from the joint is equal to shear left from the joint:
\[
\left[ \frac{\partial^3}{\partial x^3} + (2 - \mu) \frac{\partial^3}{\partial y^3} \right] \left[ w + A w_A + B w_B \right] = \\
\left[ \frac{\partial^3}{\partial x^3} + (2 - \mu) \frac{\partial^3}{\partial y^3} \right] \left[ C w_C + D w_D \right]
\]

Relation (15) can only be verified if no shear is “lost” in the load transfer device (dowel) itself. But shear could only vary along the dowel under the influence of a load acting on it and we cannot imagine the origin of such a load; hence we rely on relation (15).

5. Resolution of the boundary equations for \( x = d \)

We use next notations
\[
F = e^{-\alpha(d-a)/l} \cos \frac{d-a}{l} \beta - e^{-\alpha(d+a)/l} \cos \frac{d+a}{l} \beta
\]
\[
G = e^{-\alpha(d-a)/l} \sin \frac{d-a}{l} \beta - e^{-\alpha(d+a)/l} \sin \frac{d+a}{l} \beta
\]
\[
A = A(s)e^{\alpha d/l} \quad B = B(s)e^{\alpha d/l}
\]
\[
C = C(s)e^{-\alpha d/l} \quad D = D(s)e^{-\alpha d/l}
\]
\[
\cos(\beta d / l) = \cos \quad \sin(\beta d / l) = \sin
\]

The conditions must be valid whatever the value of the integration variable \( s \) and \( y \). Hence in writing the boundary conditions we ignore the factor
\[
(p / \pi k) \int_0^{\infty} \cos(sy / l) \sin(bs / l) \, ds
\]

The boundary equations write
\[
\delta A \cos + \delta B \sin - C \cos - D \sin = -\delta w_i
\]
\[
A \left[ s^2 (1 - \mu) \cos - \sin \right] + B \left[ s^2 (1 - \mu) \sin + \cos \right] = -M_i
\]
\[
C \left[ s^2 (1 - \mu) \cos + \sin \right] + D \left[ s^2 (1 - \mu) \sin - \cos \right] = 0
\]
\[
A \left\{ -\alpha \left[ s^2 (1 - \mu) \cos + \sin \right] + \beta \left[ s^2 (1 - \mu) \sin - \cos \right] \right\}
\]
\[
+ B \left\{ -\alpha \left[ s^2 (1 - \mu) \sin - \cos \right] - \beta \left[ s^2 (1 - \mu) \cos + \sin \right] \right\}
\]
\[
- C \left\{ -\alpha \left[ s^2 (1 - \mu) \cos - \sin \right] + \beta \left[ s^2 (1 - \mu) \sin + \cos \right] \right\}
\]
\[
- D \left\{ -\alpha \left[ s^2 (1 - \mu) \sin + \cos \right] - \beta \left[ s^2 (1 - \mu) \cos - \sin \right] \right\} = -T_i
\]

where deflection, moment and shear force in an infinite slab are for \( x = d \)
\[
w_i = \frac{F + s^2 G}{s^4 + 1}
\]
\[
M_i = \frac{1 + (1 - \mu) s^2}{s^4 + 1} G - \frac{\mu s^2}{s^4 + 1} F
\]
\[ T_s = \frac{(2-\mu)s^2}{s^4 + 1} \alpha F - \frac{(2-\mu)s^2}{s^4 + 1} \beta G \]
\[ - \frac{1-(1-\mu)s^4}{s^4 + 1} \beta F + \frac{1-(1-\mu)s^4}{s^4 + 1} \alpha G \]

By (16) and (18)
\[ C = -\delta \left[ w_i + A \cos + B \sin \right] \left[ s^2(1-\mu) \sin - \cos \right] \]
\[ D = \delta \left[ w_i + A \cos + B \sin \right] \left[ s^2(1-\mu) \cos + \sin \right] \]

By (17) and (19)
\[ A = \frac{1}{\Delta} \left\{ -s^2(1-\mu) \sin + \cos \right\} \delta w_i \left[ 2(1-\mu)s^2 \alpha + \beta \left[ 1-(1-\mu)^2s^4 \right] \right] \\
+ T_i \left[ s^2(1-\mu) \sin + \cos \right] + \alpha M_i \left[ (1+2\delta)(1-\mu) \sin - \cos \right] \\
+ \beta M_i \left[ (1-\mu) \cos + (1+\delta) \sin - \delta(1-\mu)^2s^4 \sin \right] \]
\[ B = \frac{1}{\Delta} \left\{ s^2(1-\mu) \cos - \sin \right\} \delta w_i \left[ 2(1-\mu)s^2 \alpha + \beta \left[ 1-(1-\mu)^2s^4 \right] \right] \\
- T_i \left[ s^2(1-\mu) \cos - \sin \right] - \alpha M_i \left[ (1+2\delta)(1-\mu) \cos + \sin \right] \\
+ \beta M_i \left[ (1-\mu) \sin - (1+\delta) \cos + \delta(1-\mu)^2s^4 \cos \right] \]
\[ \Delta = (1+\delta) \left\{ 2\alpha - (1-\mu)s^2 + \beta \left[ 1-(1-\mu)^2s^4 \right] \right\} \]

6. Relation between shear transfer and deflection ratio

The shear force at the right side of the joint can be expressed by
\[ Q = C \left\{ \alpha \left[ (1-\mu)s^2 \cos - \sin \right] + \beta \left[ (1-\mu)s^2 \sin + \cos \right] \right\} \\
+ D \left\{ \alpha \left[ (1-\mu)s^2 \sin + \cos \right] - \beta \left[ (1-\mu)s^2 \cos - \sin \right] \right\} \]

Replacing \( C \) and \( D \) by their values, one obtains
\[ Q = \frac{2\delta}{1+\delta} T_i \]

Hence
- when \( \delta = 0 \), \( Q = 0 \): there is no shear transfer;
- when \( \delta = 1 \), \( Q = T_i \): the full shear, generated in the infinite slab, is transferred.
7. Simplification of the boundary equations

Relation (26) allows to reduce the number of boundary equations to the next two

- Moment left from the joint is zero:

\[
\left[ \frac{\partial^2}{\partial x^2} + \mu \frac{\partial^2}{\partial y^2} \right] w + Aw_A + Bw_B = 0
\]  (27)

- Shear left from the joint is equal to a ratio of the shear in the infinite slab:

\[
\left[ \frac{\partial^3}{\partial x^3} + (2 - \mu) \frac{\partial^3}{\partial y^3} \right] w + Aw_A + Bw_B = \gamma \left[ \frac{\partial^3}{\partial x^3} w + (2 - \mu) \frac{\partial^3}{\partial y^3} w \right]
\]  (28)

where

\[
\gamma = \frac{2\delta}{1 + \delta}
\]  (29)

One shall notice that relations (27) and (28) are strictly the boundary equations of the 1993-paper.

Hence relation (29) becomes the link between the 1993 and the 1996 papers.


Ioannides (1996) recalls that for a slab resting on a dense liquid foundation and equipped with a pure-shear load transfer mechanism, following relations hold between the deflections \( w \) and the bending stresses \( \sigma \):

\[
w_l + w_u = w_f \]  (30)

\[
\sigma_l + \sigma_u = \sigma_f
\]  (31)

where \( l \) stays for the loaded side, \( u \) for the unloaded side and \( f \) for the free edge.

Applying (30)

\[
w_l + w_u = w_l + Aw_A + Bw_B + Cw_C + Dw_D = (1 + \delta)(w_l + A \cos + B \sin)
\]

\[
w_l + w_u = (1 + \delta) \left[ w_l + \frac{\delta w_l}{l + \delta} + \frac{I}{\Delta} \left[ \gamma - \alpha M_l + \beta M_l (1 - \mu) s^2 \right] \right]
\]
\[ w_i + w_u = w_i + \frac{I}{\Delta} \left[ T_i - \alpha M_i + \beta M_i (1 - \mu) s^2 \right] \]

\[ w_i + w_u = w_i + A' w_A + B' w_B \]

where

\[ \Delta' = \Delta(\delta = 0) \quad A' = A(\delta = 0) \quad B' = B(\delta = 0) \]

Hence

\[ w_i + w_u = w_f \quad (32) \]

Relation (31) can easily be verified in the same way.

If we look now for a relation between the bending stresses on both sides of the joint, we write

\[ \sigma_i = -\frac{6D}{h^2} \left[ \mu \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \left[ w_i + A w_A + B w_B \right] \]

and because \( M_x = \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} = 0 \) at the joint

\[ \sigma_i = -\frac{6D}{h^2} (1 - \mu^2) \frac{\partial^2}{\partial y^2} \left\{ w_i + A w_A + B w_B \right\} \]

\[ \sigma_u = -\frac{6D}{h^2} \left[ \mu \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \left[ C w_c + D w_d \right] \]

\[ = -\frac{6D}{h^2} (1 - \mu^2) \frac{\partial^2}{\partial y^2} \left\{ C w_c + D w_d \right\} \]

\[ = -\frac{6D}{h^2} \delta (1 - \mu^2) \frac{\partial^2}{\partial y^2} \left[ w_i + A w_A + B w_B \right] \]

because \( A, B, C \) and \( D \) are independent of \( y \).

Hence

\[ \frac{\sigma_u}{\sigma_i} = \frac{w_u}{w_i} = \delta \quad (33) \]

Relation (33) which concludes a linear ratio between the stresses and the ratio for the deflections fully disagrees with equation (45) of Ioannides (1996) which can be written as follows with our notations:
\[
\delta = \frac{[1206(a/l) + 377(\sigma_u/\sigma_v)^2 - 393(a/l)(\sigma_u/\sigma_v)^3]}{1 + 689(a/l)(\sigma_u/\sigma_v) + [370 - 154(a/l)](\sigma_u/\sigma_v)^2} \tag{34}
\]

This is surprising because in the introduction of his paper, Ioannides (1996) recalls: ‘that in his 1948 paper, Westergaard provided some preliminary considerations for the solution of the edge load transfer problem. They were, however, limited by his implicit assumption that the load transfer efficiency in terms of deflection was identical to the load transfer efficiency in relation to stress, a postulate disproved by more recent finite element investigations (Ioannides and Korovesis 1990 and 1992)...’. Our relation (33) seems to be well in agreement with Westergaard’s postulate.

In order to make some analysis and comparisons possible we establish, as Skarlatos and Ioannides, the relations for the deflections under an isolated load. We therefore go over to the limit by writing \( P = \lim_{a \to 0, b \to 0} 4pab \).

One obtains the deflection at the loaded side of the joint

\[
w_l = \frac{P}{(1+\delta)\pi kl^2} \int_0^{\infty} e^{-\alpha s/l} \cos(\beta x/l) \left[ \cos(\beta x/l) + (1-\mu) s^2 \sin(\beta x/l) \right] ds \tag{35}
\]

for a load located at a distance \( x \) of the joint.

Hence, with \( w_u = \delta w_f \)

\[
w_u = \frac{\delta P}{(1+\delta)\pi kl^2} \int_0^{\infty} e^{-\alpha s/l} \cos(\beta x/l) \left[ \cos(\beta x/l) + (1-\mu) s^2 \sin(\beta x/l) \right] ds \tag{36}
\]

and with \( w_f = w_l + w_u \)

\[
w_f = \frac{P}{\pi kl^2} \int_0^{\infty} e^{-\alpha s/l} \cos(\beta x/l) \left[ \cos(\beta x/l) + (1-\mu) s^2 \sin(\beta x/l) \right] ds \tag{37}
\]

One can verify that relation (37) for the free edge is identical with Westergaard’s relation given as (10) by Ioannides (1996).
However, with our notations, relation (1) from Ioannides for the deflection of the unloaded side becomes

\[ w_u = \frac{P}{\pi kl^2} \int_0^l e^{-\alpha s} \cos(\beta x / l) \left[ \cos(\beta x / l) + (1 - \mu)s^2 \sin(\beta x / l) \right] \frac{2}{2\alpha s^2 (1 - \mu) + \beta \left[ 1 - s^4 (1 - \mu)^2 \right]} \]

\[ x \frac{2\alpha f}{4\alpha f + 1 + 4(1 - \mu)\alpha^2 s^2 - (1 - \mu)^2 s^4} ds \]  

(38)

where \( f = q_o / kl \). The parameter \( q_o \) has dimensions of \( FL^{-2} \) and represents, following Ioannides, the force transferred across a unit length of joint per unit differential deflection across the joint.

Comparing (36) with (38) one can imagine next relation between the two

\[ \frac{2\alpha f}{4\alpha f + 1 + 4(1 - \mu)\alpha^2 s^2 - (1 - \mu)^2 s^4} = \frac{\delta}{1 + \delta} \]

However this seems is meaningless because the relation on the left side contains the varying parameter \( s \), while the term right is a constant.

It is nevertheless interesting to estimate the value of \( f \) in order to obtain identical deflections on both sides (\( \delta = l \) in our approach).

It is possible to have \( w_l = w_u = w_f / 2 \) with

\[ \frac{2\alpha f}{4\alpha f + 1 + 4(1 - \mu)\alpha^2 s^2 - (1 - \mu)^2 s^4} \approx \frac{1}{2} \]

This is only possible for very high values of \( f \), let say \( f = 10^4 \).

We then have, for usual values of \( k = 0.1 \text{ N/mm}^3 \) and \( l = 1000 \text{ mm} \), \( q_o = 10^6 \text{ N/mm}^2 \). If our interpretation is correct, this is an impossible high value for a shear stress in concrete. One must therefore conclude that the factor \( f \) represents a different assumption about load or shear transfer at the joint than expressed by our relation (15). Here lies probably the origin of the differences between stresses and deflections given by both theories.

We do not think that verification with finite elements can be helpful: it all depends on the boundary hypothesizes and thus on the initial assumptions. We think that only real measurements on a test slab can bring a better insight in the problem.

For the time being we can only perform some computations with the program “SLAB” presented together with the 1993 paper; of course they agree with relation (33): however an assumption is never verified with conclusions based on it.
In table 1, \( \gamma \) is an input value, \( \delta \) is computed by (29), \( w_f \) and \( w_l \) are computed by SLAB, \( w_u \) is computed by (30), \( w_u/w_l \) verifies boundary condition (12), \( \sigma_f \) and \( \sigma_l \) are computed by SLAB, \( \sigma_u \) is computed by (31), \( \sigma_u/\sigma_l \) verifies relation (33). The results would of course have been the same with \( \delta \) (measurable value) as input value and \( \gamma \) computed by (29).

If relation (33) is correct, the bending stress \( \sigma_l \) at the loaded side of a doweled joint submitted to a single edge load can immediately been obtained from Westergaard’s free edge formula (1948):

\[
\sigma_f = \frac{3(1+\mu)P}{\pi(3+\mu)h^2} \left[ \ln \left( \frac{Eh^3}{100ka^2} \right) + 3.84 - \frac{4\mu}{3} \right]
\]

\[
\sigma_l = \frac{1}{I+\delta} \frac{3(1+\mu)P}{\pi(3+\mu)h^2} \left[ \ln \left( \frac{Eh^3}{100ka^2} \right) + 3.84 - \frac{4\mu}{3} \right]
\]
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