Fatigue properties of plain concrete

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Summary
Based on a literature search, several types of fatigue relationships were collected, compared and analysed for plain concrete (specimens, pavements) subjected to repeated loading. A uniform fatigue relation - based on the F-S-R concept - has been proposed for the flexural tensile strength. The 50%-probability curves have been analytically described and the width of the 90%-confidence area has been quantified. This ‘new’ relationship is now being implemented into a probabilistic design method which can estimate the additional lifetime of an existing concrete pavement. This design approach is valuable for assessing airfield pavements.

Samenvatting
Met behulp van een literatuur-recherche zijn diverse vermoeiingsrelaties verzameld, vergeleken en geanalyseerd in geval van de herhaalde belasting van ongewapend beton (als proefstuk of als betonverharding). Voor de buigtreksterkte is een uniforme vermoeiingsrelatie voorgesteld, gebaseerd op het F-S-R concept. De 50%-waarschijnlijkheidscurven zijn analytisch beschreven en de wijdte van het 90%-betrouwbaarheidsgebied is gekwantificeerd. De ‘nieuwe’ vermoeiingsrelatie wordt nu geïmplementeerd in een probabilistische ontwerpmethode die een schatting geeft van de restlevensduur van een betonverharding. Deze ontwerp-aanpak is waardevol voor de beoordeling van vliegveldverhardingen.
1 Introduction
A basic question is whether new aircraft types - i.e. other and perhaps more severe load configurations - can be permitted on today’s airport pavements. Evaluations on the bearing capacity of cement concrete pavements (aprons, runways) are carried out to get a reliable estimate of their remaining lifetimes. For this purpose the designer can take account of various evaluation techniques, such as analytical models and structural performance models. FWD measurements can be used as input data for these models. The latter models comprise a fatigue strength relation of plain concrete and a cumulative damage hypothesis. It is well known that the choice of the fatigue model will strongly affect the outcome of the evaluation.

Evaluation techniques are now under study by a technical Working Group UEC (UEC = Uniform Evaluation method for Concrete pavements), managed by CROW. This paper deals with a state-of-the-art overview of fatigue models. The Working Group has proposed a uniform fatigue relation, referring to the flexural tensile strength of plain concrete and including the width of the 90% confidence interval.

2 Flexural tensile strength of concrete
2.1 Fatigue behaviour under repeated loads
In pavements, the fatigue of plain concrete due to repeated loads is governed by flexural tensile stresses. In general, the fatigue behaviour of plain concrete pavements is dependent on the types of (external) loads, their sizes and duration as well as on the number of repeated loads. The ratio between the maximum stress (serviceability state) and the static tensile strength of plain concrete (ultimate state) appears not to be a single constant.

In case of repeated loads, each cycle can initiate or extend microcracks in the concrete, provided that the related stress level exceeds a certain minimum. If ‘sufficient accumulation’ of this type of damage has occurred, material failure will occur. The higher the stress level $s_{\text{max}}$ and/or the stress amplitude ($s_{\text{min}} - s_{\text{max}}$) is, the more the concrete will be deteriorated. If a relatively high stress level is increased, failure of the plain concrete will require a smaller number of repeated loads. Failure is also exceeded by decreasing the loading frequency.

Fatigue of concrete can be simulated by means of (3-point or 4-point) bending tests on concrete beams. Then, repeated loads are performed at a constant loading frequency and for constant stress levels $s_{\text{min}}$ and $s_{\text{max}}$. The induced load-time relation has a sinus shape, which is often a rough approximation of the loading conditions in practice. The number of repeated loads to failure ($N_f$) can be empirically described by a classical Wöhler relation:

$$\frac{\sigma_{\text{max}}}{f_{\text{br}}} = A + B \log N_f$$

(1)
in which:
- $s_{\text{max}}$ denotes the maximum stress level;
- $f_{\text{br}}$ is the average static flexural tensile strength of plain concrete which is measured by static load-controlled tests (4-point bending) according to CUR Recommendation 36 (1).

Some strength values of plain concrete are presented in Table 1: $f_{\text{br}}$ represents the average flexural strength and $f_{\text{brd}}$ is the design strength exclusive of fatigue effects. According to Eurocode 2, concrete classes $C_{xx}$ refer to the characteristic compressive strength $f'_{\text{ck}}$ of plain concrete, in which $xx$ is in MPa ($= N/mm^2$). Values are determined by the statistical 5%-minimum level. Tests are conducted on 150 mm cubes at 28 days’ age. Thus:

$$f_{\text{brd}} = 0.7 \left(1.05 + 0.05 f'_{\text{ck}}\right) \text{ (MPa)}$$

(2)

The most unfavourable frequencies on concrete structures are often between 1 and 10 cycles/sec. In pavement engineering eq. (1) is mostly valid between 100 and 10 million repeated loads (for instance, heavily trafficked roads). A minor contribution of $N > 10^6$ cycles to a further growth of the total microcrack length in the concrete appears realistic for most design purposes. Several researchers report that the fatigue strength of plain concrete has a log-normal statistical distribution. The width of the confidence interval can take account of the scatter in test results, caused by variations in material properties and in test conditions. Eq. (1) can then be transformed, so that the fatigue relation refers to the characteristic strength (for instance to the 5% lower-bound values).

### 2.2 Fatigue relations under repeated loads

Many repeated load experiments were conducted on concrete prisms (axial loading) or on small beams of plain concrete (bending). Repeated loads were either tension-tension, compression-compression or tension-compression. In order to use these test results for the design of elastically supported concrete pavements under service conditions, one should also consider:

- the different response of concrete to load- or deformation-controlled circumstances;
- the so-called size-effect. The flexural tensile strength of concrete depends on the depth of the beam (or slab);
- the response of concrete as a function of the load duration. Concrete pavements can be applied by short-term as well as by long-term load configurations;
- the response dependent on the ‘load’ types. Loads can originate from vehicles, from hygrical or thermal gradients in the pavement and from differential settlements.
Formerly, fatigue relations were often based on field observations of structural deteriorations (for instance the total length of surface cracks or the percentage of cracked slabs) of in-service pavements. Later on, fatigue relations were determined under laboratory conditions. When used in FWD evaluations, the choice of the fatigue relation will be affected by the specific design model which the engineer has applied for the pavement analyses. The back-calculating process comprises of determining the mechanical parameters, e.g. the E-modulus of concrete and the modulus of subgrade reaction. Three types of fatigue relations are now presented:
- f-s relations;
- f-s-R relations;
- f-s-R-t relations.
In which f = flexural strength; s = flexural stress; R = ratio of minimum and maximum stress-es; t = load duration. Some details and limitations of the relations are also mentioned. A complete overview of all relations is gathered in a technical report of the Working Group (2).

3  F-S Relationships
The f-s relations appear the most simple formulae, based on laboratory experiments at a constant stress level and a constant loading frequency. To find the required slab thickness, the design stress $\sigma_p$ under service conditions is often implemented in the formula. This stress must take account of an extreme thermal gradient combined with the design (wheel) load on the pavement. The most well-known formulae are presented below.

3.1  Eisenmann (Germany)
The Eisenmann relation was originally derived from lab tests and refers to bending stress $\sigma_p$ from a single wheel load (3). After implementing the formula in a design method and comparing the computational results with slab behaviour in practice, the formula was slightly modified into:

$$\log N_f = 11.79 - 12.23 \frac{\sigma_p}{f_{brd}}$$

3.2  Veverka (Belgium)
The formula is used in a design method, however no reference is made to observations in practice nor to experimental results. For slab lengths less than 6 m the flexural stress $\sigma_p$ refers just to the maximum wheel load. In case of longer slabs the stress due to thermal gradients must be added (4):

$$\log N_f = 20 - 20 \frac{\sigma_p}{f_{brd}}$$

3.3  PCA (USA)
The PCA formulae are semi-empirical relations, which consider the flexural tensile stress due to a wheel load located in the slab mid position. The formulae were adapted so that wheel loads in edge position as well as thermal effects can be taken into account (5). Stress levels below 0.45 are assumed not to damage the plain concrete:

\[
\log N_t = 11.73 - 12.08 \frac{\sigma_p}{f_{brd}} \text{ if } \frac{\sigma_p}{f_{brd}} > 0.55
\]

(5a)

\[
\log N_t = \frac{4.2577}{(\frac{\sigma_p}{f_{brd}} - 0.4325)^{3.268}} \text{ if } 0.45 \leq \frac{\sigma_p}{f_{brd}} \leq 0.55
\]

(5b)

\[
\log N_t = \text{ infinite if } \frac{\sigma_p}{f_{brd}} < 0.45
\]

(5c)

3.4 Corps of Engineers (USA)

When applied to elastically supported concrete pavements, the Westergaard plate model and a multi-layer model do not yield similar deflections and / or similar flexural stresses. In case of equal deflections the Westergaard model with a Winkler foundation results into smaller stresses than a multi-layer model. These stress differences tend to increase for stiffer base structures. Rollings (6) has approximated these results in the following relations:

\[
\frac{\sigma_{\text{max}}}{f_{brd}} = 1.1 [1 - 0.09 \log N_t] \text{ (CE - Westergaard)}
\]

(6a)

\[
\frac{\sigma_{\text{max}}}{f_{brd}} = 0.8 [1 - 0.1 \log N_t] \text{ (CE - layered Elastic)}
\]

(6b)

For static loading conditions the failure mechanism of slabs is well predicted by a Westergaard-Pasternak model.
4 F-S-R Relationships

The f-s-R relations consider the minimum stress level $\sigma_{\text{min}} / f_{\text{brd}}$ due to thermal gradients and the maximum stress level $\sigma_{\text{max}} / f_{\text{brd}}$ induced by wheel loads as well as by thermal gradients. The load duration is not accounted for.

4.1 IRO-Mats-CUR (The Netherlands)

Axial fatigue tests for constant stress intervals were conducted on wet cylindrical specimens at 6 cycles per second (7). The load directions of these repeated tests were either tension-tension or tension-compression. Repeated flexural tensile loads applied at 8 cycles per second on a large number of beams ('dry' plain concrete) resulted for $\sigma_{\text{min}} / f_{\text{brd}} = 0$ into:

$$\log N_f = 14.61 - 13.78 \frac{\sigma_{\text{max}}}{f_{\text{brd}}} + 2.24 \frac{\sigma_{\text{min}}}{f_{\text{brd}}}$$ (7)

4.2 Association of the Netherlands Cement Industry (VNC)

The fatigue relation used in the Dutch design method - accepted by CROW - is a combined modification of some relations reported in the literature (2,8). This relation, implemented into VENCON which is an automated version of the Dutch design method, is according to:

$$\log N_f = \frac{16.80 (0.9 - \frac{\sigma_{\text{max}}}{f_{\text{brd}}})}{1.0667 - \frac{\sigma_{\text{min}}}{f_{\text{brd}}}} \quad \text{with} \quad \frac{\sigma_{\text{max}}}{f_{\text{brd}}} \leq 0.833 \quad \text{and} \quad \frac{\sigma_{\text{max}}}{f_{\text{brd}}} \geq 0.50$$ (8)

4.3 Other formulae

Four other formulae have been reported in (2). These refer to observed responses of test specimens or to observations of the in-service behaviour of concrete slabs.
5 F-S-R-T Relationships

The strength of a concrete specimen depends on the loading rate. The material’s response to dynamical loads is more or less governed by the deformation-rate and the creep deformation. In case of higher loading frequencies, the deformation rate will increase, causing a chance in the E-modulus and the concrete strength. Some practical values for loading frequencies are presented in Table 2. In general, the pavement engineer can ignore the loading frequency as a design parameter. Hsu (10) analyzed a large number of fatigue tests from several laboratories. He expressed the lifetime \( N_f \) as functions of the stress ratio \( R = \frac{s_{\text{min}}}{s_{\text{max}}} \), the maximum stress level and the duration of a single loading sequence (\( T \) in seconds). Especially in the case of \( R = 0 \), the response of concrete to high-cycle fatigue differs from low-cycle fatigue conditions:

\[
\frac{\sigma_{\text{max}}}{f_{\text{brd}}} = 1 - 0.0662 (1 - 0.556 R) \log N_f - 0.0294 \log T \quad (9a)
\]

\[
\frac{\sigma_{\text{max}}}{f_{\text{brd}}} = 1.20 - 0.20 R - 0.133 (1 - 0.799 R) \log N_f - 0.5530 (1 - 0.455 R) \log T \quad (9b)
\]

6 Comparison of fatigue relationships

Computed coefficients \( \frac{s_{\text{max}}}{f_{\text{brd}}} \) were calculated for two constant stress levels \( (s_{\text{min}}/f_{\text{brd}} = 0.0 \) and \( s_{\text{min}}/f_{\text{brd}} = 0.4) \) and for two numbers of repeated loads till failure \( (N_f = 10^2 \) and \( 10^6) \). These numbers of cycles appear typical for concrete pavements. Fatigue data from a literature survey have been analyzed and the results are shown in Figure 1 and in Table 3. Each fatigue relation in Table 3 includes scatter due to:
- variations in material properties;
- variations in test conditions.

7 Cumulative damage hypothesis

Several practical design methods for concrete roads and pavements (3,8) are based on a linear and cumulative damage concept for plain concrete. For each set of repeated loads with the constant minimum and maximum stress level, \( N_i = N_f \) is calculated from the Wöhler relation (1). For an actual number of load cycles \( n_i \), the damage contribution is defined as \( n_i / N_i \). By combining a total number of \( j \) sets with constant stress levels, the cumulative damage at failure is given by:

\[
\frac{1}{\sum_{i=1}^{j} N_i} = 1.0
\]
This failure criterion is according to the Palmgren-Miner hypothesis, which in fact assumes stress-induced failure by repeated loads (i.e. combined flexural tensile stresses due to traffic loads and thermal gradients). At the instant of failure a part of the slabs will show cracking patterns at the concrete surface. This simple hypothesis does not consider the sequence of loads and all loading signals are schematized as is done in Wöhler relations. However, research has shown that the Miner criterion is a sufficiently safe (conservative) design method in case of high-cycle fatigue. Moreover, the pavement design must also account for a sufficient stiffness of the slabs near transverse joints. The stiffness criterion can be based on a maximum value for the differential deflection near a transverse joint when one slab is loaded and the adjacent slab is unloaded. The calculation must incorporate a gradual softening of the slab and base due to the repetition of wheel loads which pass the slab joint.

8 Recommended revised fatigue relationship

The CROW Working Group UEC develops a probabilistic design method to estimate the additional lifetime of an existing concrete pavement. By combining a set of fatigue relations from Table 3, a statistically reliable fatigue relation (based on the F-S-R concept) was derived. Stress levels below 0.50 do not contribute to a surplus of material damage. This relation refers to plain concrete and to the loading case of repeated flexural tensile stresses:

\[ \log N_f = \frac{12903 \left(0.995 - \frac{\sigma_{\text{max}}}{f_{\text{brd}}} \right)}{1000 - 0.7525 \frac{\sigma_{\text{min}}}{f_{\text{brd}}}} \quad \text{with} \quad \frac{\sigma_{\text{max}}}{f_{\text{brd}}} \leq 0.833 \quad \text{and} \quad \frac{\sigma_{\text{max}}}{f_{\text{brd}}} \geq 0.50 \]  

(11)

This relation is indicated by ‘UEC’ in Table 3: it has a normal (statistical) distribution and refers to the 50%-probability curves. The width of the 90%-confidence area of this ‘mean’ relation corresponds to \( \log N_f = \pm 1.2 \) or \( \frac{\sigma_{\text{max}}}{f_{\text{brd}}} = \pm 0.08 \). Eq. (11) appears consistent with the static loading case for \( N_f = 10^2 \) loading cycles. A load safety factor of 1.2 is assumed in case of static loads. This corresponds well to the factor mentioned in the European design code (Table 9.2, case C in Eurocode 1) which prescribes partial safety factors which lay between 1.0 (permanent actions) and 1.3 (variable actions). Eq. (11) is equivalent to eq. (1) in case of \( A = 0.995 \) and \( B = -0.0078 \).

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References


